A new method for colour mathematical morphology based on smoothed histogram

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Abstract - This paper proposes a new method to generalise the morphological operations (particularly, dilation, erosion, opening and closing) to colour images. This method fulfils the properties established by Serra. Our method is based on a colour ordering dependent on each image, different from other methods which are based on a canonical vector ordering. Therefore, it captures the intuitiveness of what is the background on an image. We have tested our method with the basic morphological operations (dilation, erosion, opening and closing) on synthetic and real images.

1 INTRODUCTION

Mathematical morphology is a well established technique for image analysis, with solid mathematical foundations [7] that has found enormous applications in many areas, mainly image analysis, being the most comprehensive source the book of Serra [8]. A good modern introduction to mathematical morphology is [9]. Most of the practical applications are based on clever combination of a small set of operations, namely erosion and dilation, based in turn in the hit-miss transformation, or their direct derivatives, opening and closing. Mathematical morphology was initially developed for binary images and later generalised to grey-valued images [8, 10], considered as a sampled function of $\mathbb{R}^2$ in $\mathbb{R}$, or in general of any function of $\mathbb{R}^n$ in $\mathbb{R}$. Nevertheless, colour or, in general, multispectral images are samplings of function of $\mathbb{R}^n$ in $\mathbb{R}^m$, being m equal to three in the case of the usual colour images or to the number of bands otherwise. A good generalisation of morphological operations to these kinds of images would be an extremely useful and interesting goal, since it would extend the already tested techniques of image analysis based on morphology to colour images. This work is a proposal of a new method of generalisation of morphological operations to colour images.

The paper begins with a review on mathematical morphology applied to colour images. In section 3 we describe our approach and how it is formalised. Later, in section 4, we show and discuss the results of applying elementary morphological operations with our approach. Finally, in section 5 we draw the conclusions of our work.

2 COLOUR MATHEMATICAL MORPHOLOGY OVERVIEW

Serra defined the theoretical foundations of mathematical morphology and the properties that morphological operations should fulfil in [8]. These properties are:

- There should be an idea of ranking due to a sort order. Note that order, and no value, is sufficient. Real values are not used in morphology, which is based on minimum and maximum values in given sets).
- For any finite set, the ranking should yield a supremum.
- We must have the possibility of admitting an infinity of operands.

Apart from that, other desirable properties that a colour dilation should have are summarised by Köppen and coworkers in [6]:

- The colour dilation should be colour-preserving, i.e., no new colours should be introduced (as in gray level: no new level appears in a dilated or eroded image).
- The colour dilation should be an increasing operation, i. e., $\delta_B(I) \geq I$ being I any image and B any structuring element. This requires the definition of an order relationship between colour pixels, which is the key point of all this matter.
- If δ^{bin} denotes standard binary dilation, the colour dilation should be compatible with it, i.e. $\delta_B(\delta_{B'}(I)) = \delta_{\delta_B^{bin}(B')}(I)$ being B and B' structuring elements whatsoever.
- Restriction of the definition to gray scale images should become the standard gray level dilation.

The mathematical morphology was first defined for binary images, and then extended to grey-valued images. For grey scale images taking values on $\mathbb{R}$, the order relationship is established between real numbers. If we generalise the concept of image to a function taking values in $\mathbb{R}^m$ the order is not canonically defined, and this is the difficulty that all methods for generalising morphology to colour images must address. The problem of ordering a vector space, in this case $\mathbb{R}^3$, has already been treated by classical statistics, and the summary most widely cited is [2].

Literature review mentions four types of ordering: Marginal Ordering (MO), that is equivalent to ordering by one of the components; Partial ordering (PO), which uses convex-hull like sets; Conditional Ordering (CO), ordering by each component in turn; and the Reduce Ordering (RO), that performs the ordering of vectors according to a scalar function, computed from the components of each vector. The most widely used is RO, and the method we propose is also of this type.

Conner and Delp in [4] define erosion and dilation using both, MO and RO [11]. When using MO they accomplish the three conditions proposed by Serra in [8] and all of those recommended by Köppen in [6], except from the colour preservation. Although, they prove that all the basic properties are fulfilled when using RO. Nevertheless, this depends on the colour space chosen.

Chanusot and Lambert [3] propose a RO method. An integer number is assigned to each (R, G, B) colour represented by a set of three $n - bit$ numbers. The assigned number is a $3n$ bit integer obtained by mixing the three $n - bit$ numbers in a predefined order. They show that any reduced order can be expressed under this framework.

Another RO method has been studied by Angulo [1]. He defines a total order relationship using a distance to a reference colour completed with lexicographical cascades. The idea is to choose a colour space with each colour a 3-component vector. Then, choose a metric in this vector space (Euclidean, Mahalanobis or another) and finally a reference colour $\mathbf{c}_o$. Colours are ordered by its distance to $\mathbf{c}_0$ according to the chosen metrics and for equal distances, they are ordered by each of its components in turn, where the component preference is fixed in advance. This method has the advantages of great flexibility and the possibility of incorporating special orderings for components.

Finally, Hanbury in [5] uses a quite complex ordering scheme based on a physical analogy that takes into account the human side by using a perceptual system of colour coordinates, namely the (L^*, a^*, b^*) colour space. As any method based on reduced ordering, this fulfils all the desired requirements, except the compatibility with grey morphology, since no direct restriction to grey levels is possible under this charge/potential scheme. Disadvantages are those mentioned in other methods: dependency on the colour space and application of a fixed colour ordering scheme which produces unexpected results in some images.

3 A COLOUR MATHEMATICAL MORPHOLOGY METHOD BASED ON SMOOTHED HISTOGRAM

Our proposal is based on establishing a total order in the set of pixel values (colours) using a function $f : \mathbb{R}^3 \longrightarrow \mathbb{R}$, so it fits in the category of RO criteria. Current reduced ordering approaches suffer from a basic problem: they try to impose a total order based exclusively on the colours themselves, and not in the image. This is equivalent to trying to impose a canonical order to the set of colours.

In our approach, we consider the $3 - D$ colour histogram as the probability density of the appearance of colours in the image. This histogram is smoothed so that each colour exerts influence over its neighbour colours. In this way a single colour present in various tonalities or shadings would be ordered with respect to other complex colours according to its global importance. The smoothing kernel chosen is, by resemblance with Hanbury, a potential-like function. Each pixel of the image $I = \{p_1 \dots p_N\}$ is considered as a unit charge in the colour space, which will be quantised as stated before, and so it is $Z \times Z \times Z$. The potential created by pixel p_i with colour $\mathbf{c}_i = (c_i^1, c_i^2, c_i^3)$ at any point $\mathbf{c}_j = (c_j^1, c_j^2, c_j^3)$ of the colour space will be

$$V_{ij} = \begin{cases} 1 \text{ if } r_{ij} = 0 \\ \frac{1}{r_{ij}^d} \text{ if } r_{ij} \neq 0 \end{cases}$$

where r_{ij} is the distance between both colours using a suitable metrics; in our case Mahalanobis distance. We have chosen the (R, G, B) space; and d is a smoothing parameter $d \geq 1$ to be chosen. Notice that, since the colour space $Z \times Z \times Z$ is quantised, $r_{ij} \geq 1$ and so $\frac{1}{r_{ij}^d} \leq 1$.

Once all charges have been placed, potential at any point of the colour space is calculated as

$$V_j = \sum_{i=1, i \neq j}^{N} V_{ij}$$

A reasonable way to choose the parameter d is to place a unit charge at the centre of the admissible $3 - D$ cube (or shape) of the colour space and make its potential at the furthest border equal to just a negligible amount (1%) of the potential at the charge (which is 1). For our case the (R, G, B) space used was quantised to 100 parts in each component and so a reasonable value for d was 2.

The points in the colour space are simply ordered by their potential V. A possible problem is that the application defined by this potential might not be injective, so it is said, two or more different colours may have the same potential. This is not a problem for the application of morphological operations, as long as two pixels with these colours do not belong to the translated of the structuring element $B_{\mathbf{x}}$ for any point $\mathbf{x}$ in the image. If this happens, one of them is randomly chosen (once selected it must be the same for all points $\mathbf{x}$ where this happens). This possibility leads to some comments on implementation: the potential does not have to be calculated for every point of the colour space, only for those, which appear in the image. Its potentials are stored in an ordered linked list of records with two fields: the potential itself and a pointer to a list of three-component colours to which it is assigned. Usually, this list has just one element; if not, colours are stored together with the image coordinates of where they appear. It makes possible to choose the appropriate colour at any location $\mathbf{x}$.

This definition fulfils the required conditions defined by Serra in [8], and even the desirable conditions defined by Köpen in [6]. These conditions are:

- There is total order (a supreme for each finite set)
- Operations can be applied iterativele
- Erosion and dilation are colour-preserving (no new colour is introduced)
- Dilation is increasing, according to the colour ordering defined for that image
- Colour dilation is compatible with the binary dilation, since it is a direct generalisation of grey level, which it is.

4 RESULTS

A problem that is always present in work on colour morphology is the choice of an appropriate evaluation criterion. As far as we know, all the published results rely on subjective visual evaluation. This is because, even if a suitable metric between colour images (Euclidean, perceptual, etc.) exists, there is no canonical result to compare with which the obtained images. We think this is a difficult problem for which we have no obvious solution so we are also obliged to use subjective evaluation.

The experiments consist of the application of the basic morphological operators (erosion, dilation, opening and closing) with a disk-shaped structuring element on several images chosen to illustrate the advantages and inconveniences of the proposed method. The first image is a map of the Valencia Community (east of Spain) with very few colours and a very simple structure. The next is an image of a painting from Joan Miró that is composed mostly by planar patches, but also contain thin lines and graded tonalities. The last example is a real image of a mosaic dragon (Güell Park, Barcelona) with several colours and a natural background similar to the one used by Hanbury [5]. All the experiments are done in (R, G, B) colour space, and the Mahalanobis distance has been used to apply the histogram smoothing kernel.

In figure 1, we can observe that the blue of the sea is considered the background; the yellow of the land and the dark blue coastline dominate on it, so an erosion decreases the size of the land. It also eliminates small details like letters and lines, whose colours are less abundant, as expected from an erosion. Conversely, the dilation creates a continuous line from the dashed lines and highlights the demarcation lines. Our method works very well in this image, which has only 32 different colours.

The Miro's picture (see figure 2) has more different colours than the Valencia map. The white background is the most abundant colour and the greenish almost white tonality in the lower left corner contributes to the potential of the white background, so erosion eliminates the thin lines, independently on over which variation of the background they run. Note the annulment of Miró's signature in the lower right corner after an erosion or opening.

The last example (figure 3) shows the application of our method to a natural image, the dragon sculpture in the Güell Park (Barcelona). In this case the grey tonalities play the role of background and, in general, an erosion extends this background over coloured patches

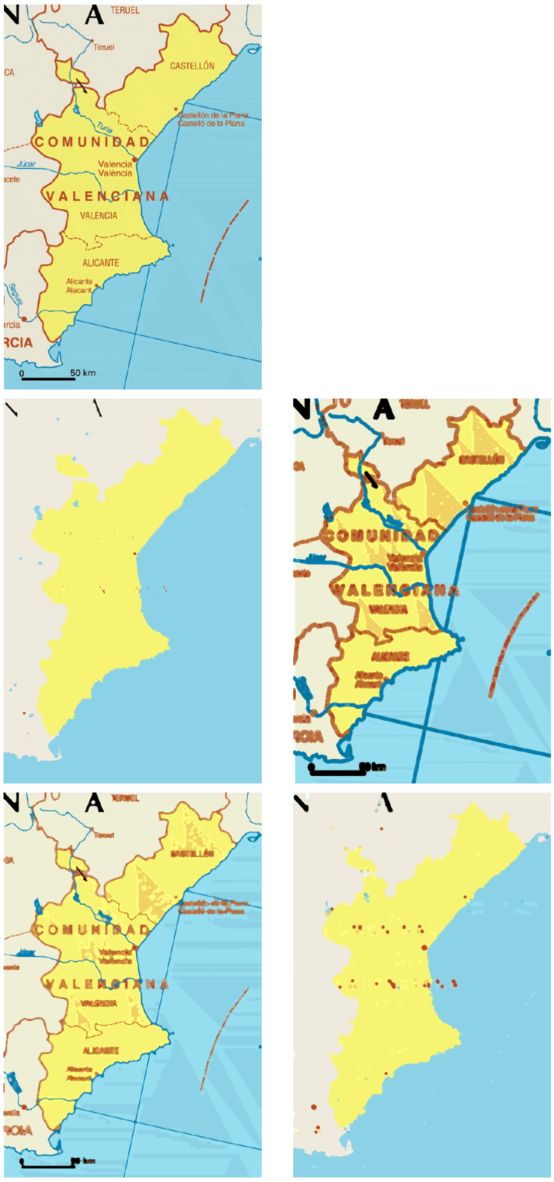

Figure 1: Valencia original map and erosion, dilation, opening and closing from left to right and up to down with a disk of radius 3 pixels. The RGB colour space and Mahalanobis distance has been used.

on the dragon's body. The opening makes these patches larger and more uniform than in the original image.

5 CONCLUSIONS

This paper has presented a method to generalise the basic morphological operations to colour images. We have reviewed previous proposals on colour ordering for mathematical morphology operations. We see that previous methods are based on canonical ordering of the vector colour space. These methods suffer from a fundamental drawback: they cannot capture the intuitiveness of the expected results for every possible image. Consequently, we renounce to try this approach and, on the contrary, give a method to construct a concrete colour ordering for each particular image. The basic idea of morphology (to detect and analyse objects on a background) gives us the clue to this ordering: the most abundant colour should be considered background, and the importance of the colours is measured inversely to its frequency of appear-

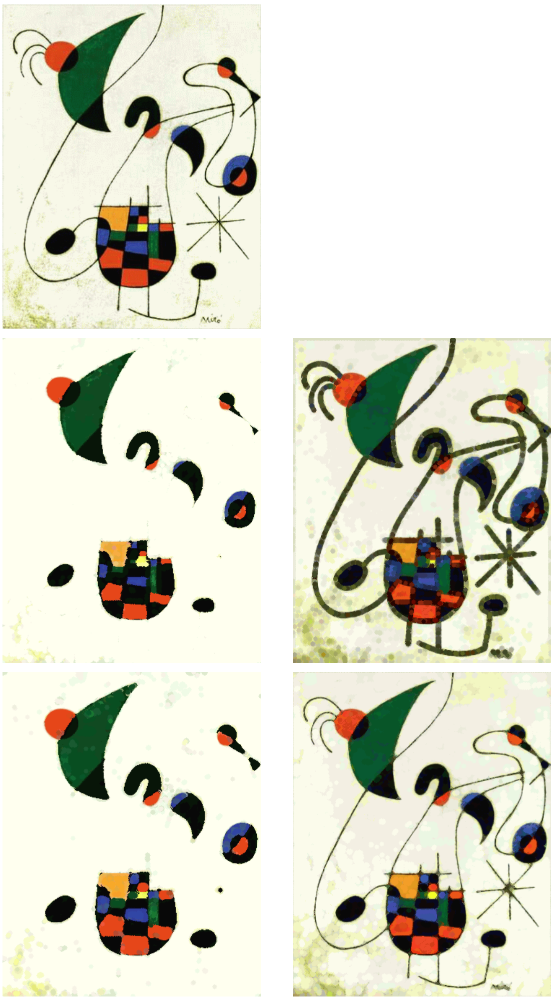

Figure 2: Miro picture's original image and erosion, dilation, opening and closing from left to right and up to down with a disk of radius 3 pixels. The RGB colour space and Mahalanobis distance has been used.

Figure 3: Original Drac picture and erotion, dilation, opening and closing with a disk of radius 3 pixels. The RGB colour space and Mahalanobis distance has been used.

ance. Our approach considers the 3-D colour histogram as the probability density of the appearance of colours in the image, and to smooth it so that each colour exerts influence over its neighbour colours.

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